PRE-CALCULUS STANDARDS

Topics 1 – 8 are appropriate for January contests
Topics 1 – 11 are appropriate for February contests
Topics 1 – 14 are appropriate for contests in March and April

For any month, only the standards as listed may be used.

1. **Demonstrate an understanding of the theory of functions.**
   - find domains; ranges; and specific values of functions in functional notation.
   - given two functions perform the algebra of functions including composition of functions.
   - determine if a given function is:
     - a. symmetric (with respect to the axes and/or origin.
     - b. periodic
     - c. monotonic
     - d. bounded
     - e. continuous
   - identify and graph polynomial and rational functions and determine asymptotes.
   - define and use parametric forms of functions and convert from parametric to Cartesian form.
   - given a function; determine the inverse and state whether or not the inverse is a function.

2. **Demonstrate an understanding of connection between circular and trigonometric functions and their inverses.**

   - evaluate circular and trigonometric expressions involving any of the six functions and their inverses.
   - given the equation for a circular (trigonometric) function; identify and/or sketch the graph and the graph of its inverse relation and state the domain and range of the original
   - function and its associated inverse function.
   - identify its equation when given a graph of any of the six circular functions.
   - state the period; amplitude; phase shift; and vertical shift of a circular function and/or graph of the function.

3. Demonstrate an understanding of the trigonometric identities.

   - prove that a given trigonometric equation is an identity by applying the Pythagorean relation and reciprocal identities.
   - prove that an appropriate trigonometric equation is an identity when given the sum and difference formulas for the cosine; sine; and tangent.
   - prove that an appropriate trigonometric equation is an identity when given the double order formulas for sine; cosine; and tangent.
   - prove that an appropriate trigonometric equation is an identity when given the half-angle formulas for sine; cosine; and tangent.

4. **Demonstrate the ability to apply trigonometry to problem solving situations.**
   - solve a right triangle given two sides; or a side and an acute angle.
   - use the appropriate trigonometric function(s) to solve problems involving right or oblique triangles.
   - apply the Law of Sines.
   - apply the Law of Cosines.
   - find the area of an oblique triangle.
   - estimate the solution to a problem involving a right or oblique triangle.
   - in the SSA case determine whether 0; 1; or 2 triangles exist and determine the
   - triangles (if they exist)

5. **Demonstrate the ability to solve a variety of trigonometric (circular) equations**.
   - find the general solutions to a trigonometric equation
   - find particular solutions to a trigonometric equation within a given domain.
   - solve equations involving inverse of circular/trigonometric functions.

6. **Demonstrate an understanding of conic sections and loci.**

   - given the description of a locus determine the equation of the locus.
   - given the equation of a line determine slope and y-intercept; and graph the line.
   - given the equation of a circle determine the center and radius; and graph it.
   - given the equation of a parabola determine vertex; focus; and directrix; and graph it.
   - given equation of an ellipse in standard form; determine the center; foci; and vertices; graph it.
   - given the equation of a hyperbola in standard form; determine the foci; vertices; and asymptotes; and graph it.
   - determine new equations resulting from translation or rotation of axes.
   - identify the graph of any second-degree equation.
   - express a quadratic equation in general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and use $B^2 - 4AC$ to distinguish conics.
   - recognize degenerate and imaginary cases.

7. **Demonstrate an understanding of the relationship between exponential and logarithmic functions and their application to problem situations.**

   - evaluate expressions involving rational exponents.
   - sketch the graphs of exponential functions and logarithmic functions of different bases.
   - solve equations involving exponential functions and logarithmic functions.
   - solve real-world problems involving exponential functions and logarithmic functions.
   - simplify expressions using the relationships between logarithms and exponents.
   - express the number e and the expression 'e to the x' as infinite series

8. **Demonstrate the ability to solve problems using concepts from matrix algebra.**

   - apply determinants to solve systems of equations.
   - invert a square matrix.

9. **Demonstrate the ability to solve problems using vectors.**

   - find a vector in standard position equal to a given vector.
   - determine magnitude and direction of vectors.
   - identify perpendicular and parallel vectors.
   - determine the measure of the angle between two vectors.
   - resolve a vector into component vectors.
   - add and subtract vectors and multiply a vector by a scalar.
   - find the dot product of two vectors.
   - use vectors to solve real world problems.

10. **Demonstrate an understanding of polynomial and rational functions; their parametric equations and their graphs.**

    - given a polynomial function determine intercepts and sketch the graph.
    - given an equation of rational function determine intercepts and asymptotes and sketch the graph.
    - given a set of parametric equations sketch the graph.

11. **Demonstrate an understanding of graphs in the polar coordinate system and their relation to the Cartesian coordinate system.**

- graph points in the polar coordinate system.
- convert between polar coordinates and Cartesian coordinates.
- express complex numbers in polar or trigonometric form.
- convert equations in polar form to Cartesian form.
- convert equations in Cartesian form to polar form.
- graph polar equations and identify specific types (roses; limaçons; spirals; and conics)
- use de Moivre's theorem to find powers and roots of complex numbers.

***************END OF STANDARDS FOR FEBRUARY***************

12. **Demonstrate understanding of mathematical induction and sequences and series.**

- given an expression of rule for the nth term find any term of the sequence.
- given a sequence find a formula for the nth term in the sequence.
- find the nth term of a binomial expansion.
- find the sum of an arithmetic series.
- find the sum of a finite or infinite geometric series if it exists.
- define convergent and divergent sequences and series, determine limits if they exist.
- determine whether a sequence is increasing or decreasing.
- find the least upper bound and greatest lower bound of a sequence if they exist.
- express a series in sigma notation.
- use mathematical induction to prove series formulas.
- use mathematical induction to prove inequality formulas.

13. **Demonstrate the ability to solve problems using probability and statistics.**

- find probabilities of simple events.
- find probabilities using Venn diagrams.
- find probabilities of mutually exclusive events.
- find probabilities of independent events.
- define an event and/or the complement of an event.
- find probabilities of the complement of an event.
- find conditional probabilities.
- find probabilities in binomial distributions.
- determine a standard (z) score in a normal distribution.

14. **Demonstrate an understanding of the concept of limits and its applications.**

- geometrically illustrate functions for which x increases without bound and find limits, if they exist.
- find when possible for any neighborhood of a number L; a neighborhood of a point a such that f(x) is in the neighborhood of L when x is in the neighborhood of a.
- calculate limits of functions using theorems about limits.
- geometrically illustrate functions which are continuous at a point and/or continuous on an interval.
- given a rational function f(x) find the limit if it exists at a point of discontinuity.
- using the definition of the derived function of f(x) find the derive function.
- determine the equation of tangents to graphs of curves given the slope formula.

***************END OF STANDARDS FOR MARCH***************